Inception Point Relationship for Flat-Sloped Stepped Spillways

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Abstract. Many small earthen embankments are faced with hazard classification changes due to urban encroachment. As a result, some embankments have inadequate spillway capacity. To bring the dam into compliance with state and federal dam safety laws, rehabilitation of the dam is often required. RCC stepped spillways are becoming a popular choice for addressing these issues. However, design guidelines for RCC stepped spillways applied to small earthen dams are scarce, especially for spillways with slopes flatter than 2(H):1(V).

A two-dimensional, physical model was constructed to evaluate the air entrainment inception point location in a 4(H):1(V) stepped spillway. Step heights of 38 mm, 76 mm, and 152 mm were evaluated. The physical model was constructed with a broad-crested weir, and model unit discharges ranging from 0.11 $m^3/(s·m)$ to 0.82 $m^3/(s·m)$ were tested. The research findings show that Chanson's relationship effectively predicts the location of the inception point for slopes as flat as 4(H):1(V) for Froude surface roughness values (F_*) greater than 10, which in this study corresponds to model step heights of 38 mm and 76 mm. Chanson's relationship did not adequately predict the location of the inception point for F_* less than 10 which for this study corresponds to a model step height of 152 mm. A new relationship for predicting the location of the inception point was developed from this data set, and it is applicable for flat sloped stepped spillways with F_* ranging from 1 to 100. This relationship is similar to Chanson's, but it is optimized for flat-sloped stepped spillways with a broad-crested weir crest section. This research is expected to assist engineers with the design of stepped spillways applied on relatively flat embankment dams.

Keywords. Dam rehabilitation, energy dissipation, inception point, hazard classification, RCC, stepped spillway, and flooding.

Introduction

Roller compacted concrete (RCC) stepped spillways are becoming a popular choice in the rehabilitation of small earthen embankment dams. Earthen embankments located in highly urbanized areas are typically faced with hazard classification changes that often require an increase in spillway capacity. Modifying the existing dam or spillway dimensions are not always viable options due to land right restrictions, topography, and/or changes in land use. Consequently, RCC spillways are chosen because of 1) construction cost savings (i.e. steel reinforcement not required and typically shorter construction time) and 2) the energy dissipation is significant resulting in a smaller stilling basin, an additional cost savings for the project (Portland Cement Association, 2002).

Engineers are often challenged with the design of these structures due to the lack of literature on air entrainment and energy dissipation for stepped spillways applied to small embankment dams. Air entrainment is expected to develop in stepped spillways and may cause a flow bulking, an increase in flow depth as a result of air in the flow. Consequently, air entrainment may have a direct impact on the design height of the training walls. Additionally, incoming velocities and energy dissipation are necessary components in designing the stilling basin. Yet, most energy dissipation results reported in literature are downstream of the inception point, the location where the turbulent boundary layer reaches the free surface. For a small embankment dam, a RCC spillway has a relatively short chute length such that air entrainment may not fully develop in the spillway chute. Consequently, the energy dissipation and velocities upstream of the inception point become vitally important in stilling basin design. The scope of this paper is to 1) examine the observed inception point location within a stepped spillway chute applied to embankment dams, 2) evaluate the effect step height has on the inception point, and 3) determine whether relationships developed by other researchers are applicable for estimating the distance from the spillway crest to the inception point.

Background

As illustrated in Figure 1, the inception point in stepped spillway chutes as defined by Chanson (1994) is the location where the turbulent boundary layer reaches the free surface. Upstream of this point, the water has a glassy, smooth appearance. The flow becomes frothy as air entrainment becomes more fully developed downstream of the inception point. Air entrainment creates a flow bulking, an increase flow depth as a result of air in the flow. Most design criteria require that the training walls be tall enough to contain the design flow. Following these criteria, the aerated flow region in stepped spillways is expected to have taller training walls than in non-aerated flow regions.
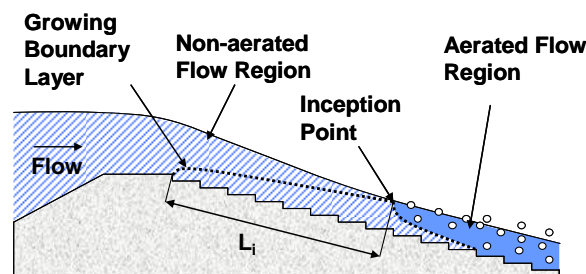

Figure 1. Schematic of the inception point in relation to the stepped spillway.

A relationship for determining the distance between the spillway crest and the inception point was first proposed by Wood et al. (1983).

$$x = 13.6(\sin\theta)^{0.0796}\left(F_*\right)^{0.713}k_s \tag{1}$$

where x = the distance along the profile from the crest, θ = channel slope, F_* = Froude number defined in terms of the roughness height: $F_* = q/[g\,(\sin\theta)k_s^3]^{0.5}$, q = unit discharge, g = gravitational constant, and k_s = the surface roughness (Wood et al. 1983). This relationship covered a range of chute slopes, roughness, and discharges. The relationship was later enhanced by Chanson (1994, 2002) for application in stepped spillways.

$$L_{i*} = 9.719(\sin\theta)^{0.08}\left(F_*\right)^{0.713}k_s \tag{2}$$

where L_{i*} = distance from the start of growth of boundary layer to the inception point of air entrainment, θ = channel slope, F_* = Froude number defined in terms of the roughness height: $F_* = q/[g\,(\sin\theta)\,\{k_s\}^3]^{0.5}$, q = unit discharge, g = gravitational constant, k_s = h cos(θ), and h = step height (Chanson, 1994). Like Wood et al. (1983), Chanson's (1994) relationship takes into account the channel slope, θ and Froude surface roughness, F_*. The difference between the relationship developed by Wood et al. (1983) and Chanson (1994) is that Chanson's relationship takes into account step height in the calculation of F_* and the surface roughness, k_s. Chanson (1994) further enhanced his relationship by conducting a sensitivity analysis such that the coefficient 13.6 in Wood et al. (1983) relationship changed to 9.719. Chanson (1994) based his relationship primarily on model and prototype data for stepped spillways having steep chute slopes ($\theta \geq 22°$). Chanson (2002) cautions the use of the equation outside the range for which it was developed.

Boes and Hager (2003) also developed a relationship to determine the distance between the start of the turbulent boundary layer and the inception point.

$$L_I = \frac{5.90h_c^{6/5}}{(\sin\theta)^{7/5}h^{1/5}} \tag{3}$$

where h_c = critical depth and all other parameters were previously define. This relationship was based on some of the same data used by Chanson (1994). Like Equation 2, Equation 3 is written in terms of θ, F_*, and h, and it's applicable for chute slopes ranging from 26° to 75°. The primary difference between the two equations is the definition of inception point. Chanson (1994) defines the inception point as the location where the turbulent boundary layer reaches the free surface whereas Boes and Hager (2003) define the inception point as the location where the air concentration on the pseudobottom of the chute is approximately 1%. Hunt and Kadavy (2008b) observed significant disturbance at the free surface upstream of air concentrations reaching 1% at the pseudobottom of the chute for stepped spillways having a 4(H):1(V) slope. This observation indicates that Boes and Hager's (2003) relationship is likely to over predict the location where flow disturbance occurs on the free surface.

Stepped spillways applied to embankment dams are expected to be relatively flat ($\theta \leq 22°$) with short chute lengths. Most literature provides information regarding energy dissipation and velocities in stepped spillways downstream of the inception point. Hunt and Kadavy (2008a and 2008b) have indicated that energy dissipation in a stepped spillway chute is determined differently depending on whether one is upstream or downstream of the inception point. Energy dissipation is a component in determining the dimensions of the stilling basin, so the inception point becomes vitally important in the design as well. Even though the inception point relationships presented in Equations 1 and 2 are applicable for steep slope ($\theta \geq 22°$) stepped

spillways, they provide a basis for evaluating the inception point location for flatter slopes. This research is expected to provide additional information with regards to air entrainment and energy dissipation in stepped spillways with slopes less than 22°.

Experimental Set-up

A two-dimensional 1:8 scale model of a stepped spillway was constructed for a specific study on a proposed stepped spillway for Renwick Dam in North Dakota (Hunt and Kadavy, 2008a and 2008b). This model served the purpose for this generalized study to evaluate the effects steps have on the inception point location in a 4(H):1(V) stepped spillway. The model development was based on Froude similitude. It is recognized that viscous forces and surface tension cannot be ignored in highly air entrained flows. If ignored, these forces can create scale effects, and the scale effects may cause data to be misinterpreted. Scale effect is a term used to describe distortions that are introduced by ignoring secondary forces (i.e. viscous forces and surface tension). Chanson (2002) recommends a model scale larger than 10:1. Boes and Hager (2003) propose a minimum Reynolds number of 10^5 and a minimum Weber number of 100. Taksahasi, et al. (2006) recommend that Froude, Reynolds, and Morton numbers similarity be satisfied, but they recognized this can only be achieved at full-scale.

Figure 2 illustrates the schematic of the 2-D model used during the test. The stepped spillway was constructed with a broad crested weir with the downstream edge of the weir corresponding to station 0.0 m and step 0. The spillway slope is 4(H):1(V). Step heights of 38 mm (1.5 inches) were originally tested under a series of flow conditions as summarized in Table 1. Table 1 also summarizes the Reynolds number, average velocity at the inception point, and the Weber number. The Reynolds and Weber numbers are well within the recommended guidelines set forth by Boes and Hager (2003). After testing the 38 mm (1.5 inches) steps, every other step was removed to create step heights of 76 mm (3.0 inches). These steps were tested under the same flow conditions as the first series. Upon the completion of this testing, 152 mm (6.0 inches) high steps were created by removing every other 76 mm (3.0 inches) steps and tested. This experimental set-up subsequently allowed three different step heights to be evaluated to determine the impact the steps had on the inception point.
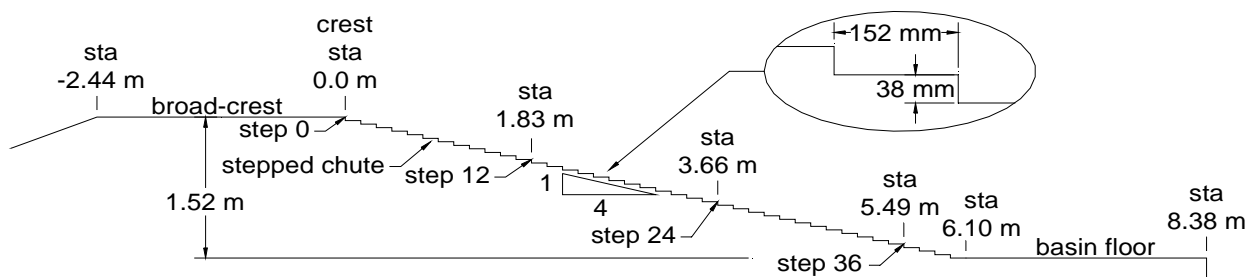

Figure 2. Schematic of stepped spillway model.

Table 1. Summary of model unit discharge (q), Reynolds number (R), average velocity (V) at inception point, and Weber numbers (W).

q (m³/(s·m))	R	V (m/s)	W
0.11	9.50E+04	2.42	112
0.20	1.81E+05	3.08	142
0.28	2.50E+05	3.54	163
0.42	3.74E+05	4.02	186
0.62	5.49E+05	4.71	218
0.82	7.33E+05	5.06	234

The model was constructed across the full width of a 1.8 m (6 ft) wide flume. The flume walls were 2.4 m (8 ft) tall, and the vertical drop of the spillway model was 1.5 m (5 ft). A moveable carriage set atop rails on the flume walls as shown in Figure 3 allowed manual point gage readings of the water and bed surfaces to be collected. Cross-sectional velocity profiles along the broad crested weir were collected with an acoustic Doppler velocimeter (ADV) that was attached to the carriage point gage. The velocities and flow depth measurements at the broad-crested weir were used to develop a calibration curve of the flow versus upstream head. This carriage was also used in velocity measurements along the crest section. A second set of rails were attached to the inside of the flume walls and set parallel to the chute slope. These rails were used to collect velocity profiles normal to the spillway chute along the centerline.

Figure 3. Data collection using the mobile carriage along the top of the test flume walls.

During each test, photographic observations recorded the inception point. The inception point was noted as the location where the flow changed from a glassy, smooth appearance to a more frothy appearance at the free surface. The inception point location was described quantitatively by its horizontal distance from the spillway crest and with regards to step location within the spillway chute. This information was translated to a corresponding L_i as defined in Figure 1.

Results and Discussion

The inception point location in stepped spillway chutes provides vital information with regards to designing chute training walls and the downstream stilling basin. Upstream of the inception point the flow is expected to be calm and smooth. Downstream of the inception point, an aerated flow region is expected. When fully developed, the aerated flow region becomes frothy at the free surface such that white water is observed. As a result, bulked flow, an increase in

flow depth as a result of air in the flow, is expected. Many design criteria require the training walls to be tall enough to contain the design discharge, and the knowledge of where the inception point occurs in the spillway chute will allow design engineers to make the judgment in whether to factor the air entrainment in the design of these walls.

According to Hunt and Kadavy (2008a and 2008b), the energy dissipation upstream and downstream of the inception point behaves differently. Hunt and Kadavy (2008a) found that the energy dissipation increases linearly from 0 to 30% upstream of the inception point. Downstream of the inception point, the energy dissipation increases in a more logarithmic trend (Hunt and Kadavy, 2008b). Having the knowledge of where the inception point is located in the spillway chute provides engineers the tools necessary for determining the energy dissipation in the spillway. This energy dissipation is a key component for designing the stilling basin.

Throughout this study, photographs of the flow were taken to capture the changes in the spillway flow appearance as it descended the chute. Figures 4a, 4b, and 4c illustrate the changes in the flow appearance due to the effect of changing step height. The discharge for each of these figures was 0.28 m^3/(s·m), and the step heights illustrated in Figures 4a, 4b, and 4c are 38 mm (1.5 inches), 76 mm (3.0 inches), and 152 mm (6.0 inches), respectively. As the flow overtops the spillway, the flow has a smooth, glassy appearance. As the turbulent boundary layer nears the free surface, the flow showed a minor undulating pattern. The turbulent boundary layer was noted to reach the free surface when the flow became more erratic in behavior such that it appeared frothy at the surface.
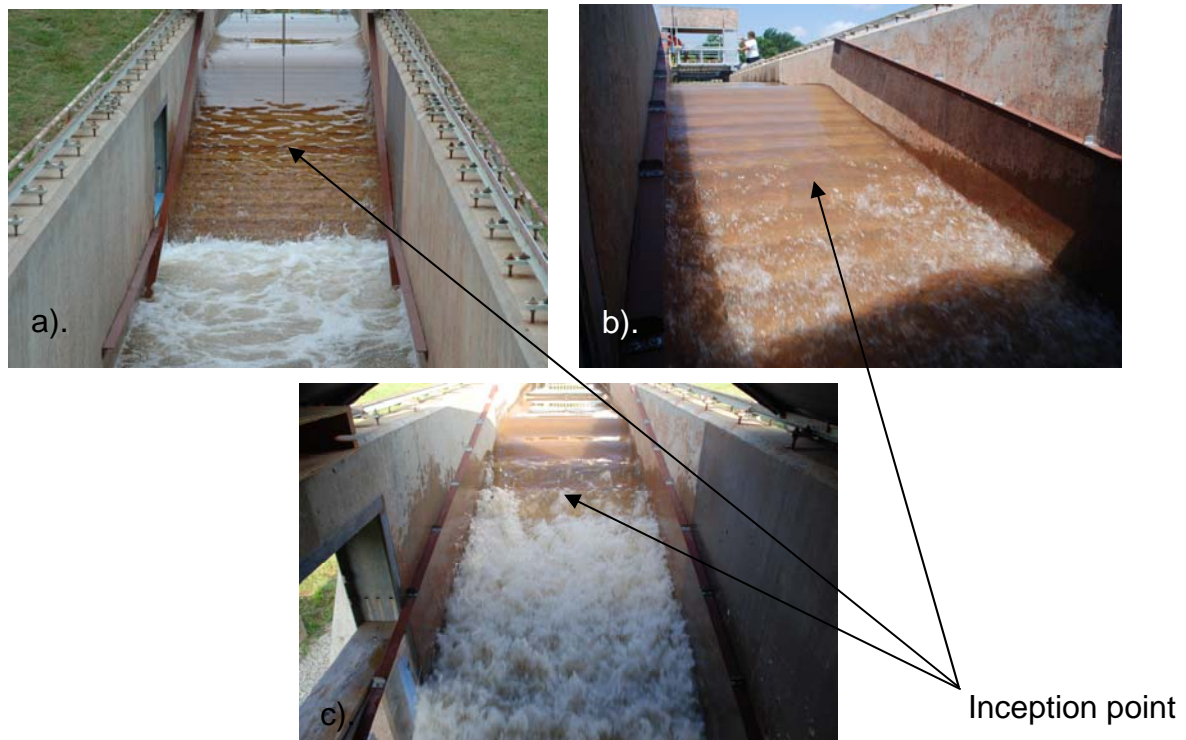

Figure 4. **Inception points for a 0.28 m^3/(s·m) flow overtopping 4(H):1(V) stepped spillway with step heights a) 38 mm (1.5 inches) steps, b) 76 mm (3.0 inches) steps, and c) 152 mm (6.0 inches) steps.**

Figures 5a, 5b, and 5c illustrate a comparison of a stepped spillway having step heights of 152 mm (6 inches) while tested under different flow conditions. The discharges shown in Figures 5a, 5b, and 5c are 0.11 m^3/(s·m), 0.28 m^3/(s·m), and 0.62 m^3/(s·m), respectively. As the

discharge increases, the inception point was noted to move further downstream of the spillway crest. At lower discharges, the inception point was more defined than at higher discharges. Higher discharges often had a more minor undulating pattern leading up to the frothy white water appearance at the surface. This minor rippling was thought to be the turbulent boundary layer approaching the free surface.
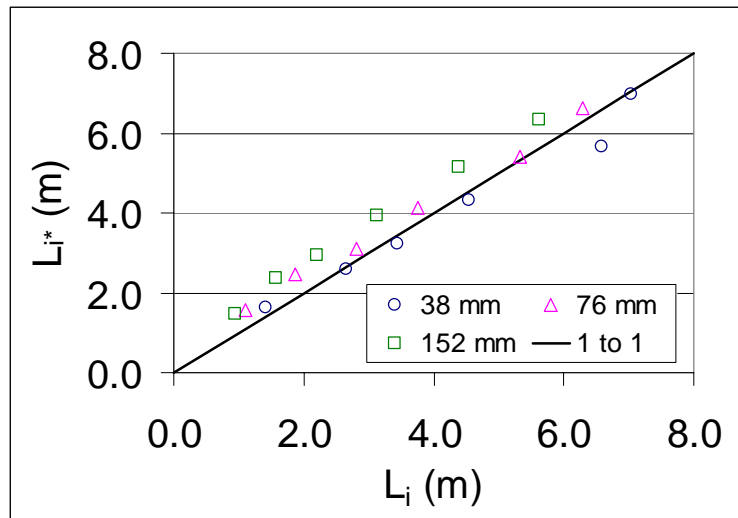

Inception Point

Figure 5. Inception points for a 4(H):1(V) stepped spillway having 152 mm (6.0 inches) step height for three flowrates a) 0.11 m³/(s·m), b) 0.28 m³/(s·m), and c) 0.62 m³/(s·m)

Table 2 summarizes the observed distance, L_i, from the downstream edge of the crest to the inception point with regards to unit discharge, q; step height, h; surface roughness, $k_s = h*\cos(\theta)$; and Froude surface roughness, F_*. Also, summarized in Table 2 is the predicted L_{i*}. Equation 2 was used to calculate L_{i*}. Table 2 and Figure 6 provide direct comparison of the observed L_i and calculated L_{i*}. The percent difference between L_i and L_{i*} ranged from 2% to 50%. As the step height increased, the difference between L_i and L_{i*} also increased. Additionally, Figure 6 illustrates that Chanson's relationship appears to more closely predict the distance from the crest to the inception point for the two smaller steps. An inspection of Table 2 reveals that step heights of 38 mm (1.5 inches) and 76 mm (3.0 inches) correspond to F_* ranging from 10 to 100 while a step height of 152 mm (6.0 inches) corresponds to F_* less than 10. It may be concluded based on these results that Chanson's relationship may be applied when the range of F_* is between 10 and 100. Caution should be used in the application of Equation 2 outside this range.

Table 2. Summary of unit discharge, step height, Froude surface roughness, inception point.

q (m^3/(s·m))	h (mm)	k_s (m)	F_*	L_i (m) observed	L_{i*} (m) Chanson (1994)	L_i/k_s observed	L_{i*}/k_s Chanson (1994)
0.11	38	0.037	9.8	1.41	1.63	38.1	44.2
0.20	38	0.037	18.6	2.65	2.58	71.6	69.8
0.28	38	0.037	25.7	3.44	3.25	93.0	87.8
0.42	38	0.037	38.3	4.54	4.32	122.7	116.9
0.62	38	0.037	56.3	6.59	5.68	178.2	153.6
0.82	38	0.037	75.2	7.05	6.99	190.6	189.0
0.11	76	0.074	3.5	1.09	1.55	14.8	21.0
0.20	76	0.074	6.6	1.87	2.47	25.3	33.4
0.28	76	0.074	9.1	2.82	3.11	38.1	42.0
0.42	76	0.074	13.5	3.76	4.12	50.8	55.7
0.62	76	0.074	19.9	5.32	5.41	72.0	73.2
0.82	76	0.074	26.3	6.28	6.61	85.0	89.4
0.11	152	0.148	1.2	0.94	1.48	6.3	10.0
0.20	152	0.148	2.3	1.57	2.36	10.6	15.9
0.28	152	0.148	3.2	2.20	2.94	14.9	19.9
0.42	152	0.148	4.8	3.12	3.94	21.1	26.6
0.62	152	0.148	7.1	4.39	5.17	29.7	34.9
0.82	152	0.148	9.4	5.62	6.34	38.0	42.9

Figure 6. Predicted L_{i*} versus observed L_i for a 4(H):1(V) spillway chute with step heights of 38 mm (1.5 inches), 76 mm (3.0 inches), and 152 mm (6.0 inches).

The data was further examined to determine if a new relationship could be developed to more accurately predict the location of the inception point. For this analysis, the observed L_i normalized by the surface roughness, k_s, was plotted against the Froude surface roughness, F_*. Figure 7 reflects this analysis. The findings show that the relationship by Chanson (1994) begins to break down as F_* becomes smaller than 10. A power fit, as provided in Equation 4,

was applied to the data and found to fit with a coefficient of determination, R^2, near one. As a result, Chanson's relationship as provided in Equation 2 was modified such that 1) the exponent to $\sin(\theta)$ was held constant at 0.08, 2) the exponent 0.713 to the F^* was replaced by the power fit exponent of 0.86, and 3) the constant 9.719 was varied. The exponent to $\sin(\theta)$ was held constant because of the single chute slope examined in this study. This analysis yielded a new relationship, Equation 5, for predicting the inception point in 14° sloped stepped spillways having a broad crested weir.

$$L_i = 5.38(F_*)^{0.86} k_s \qquad (4)$$

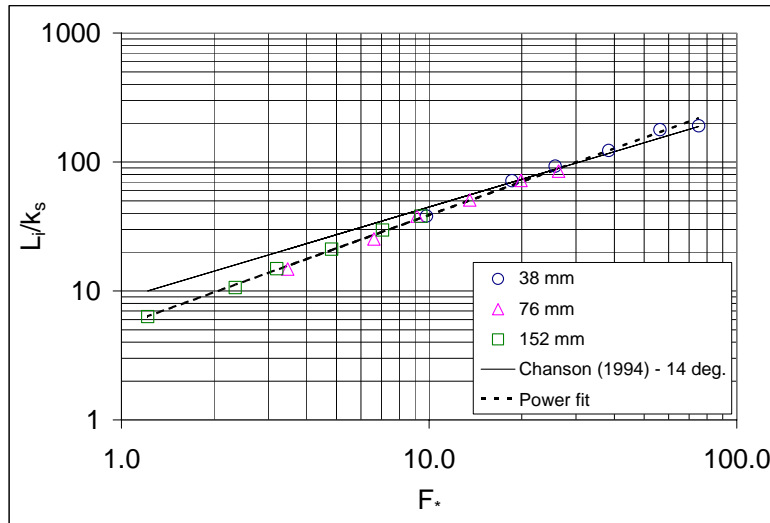$$L_i = 6.10(\sin\theta)^{0.08}(F_*)^{0.86} k_s \qquad (5)$$

Figure 7. Normalized L_i versus Froude surface roughness, F_* for a 4(H):1(V) spillway chute with step heights of 38 mm (1.5 inches), 76 mm (3.0 inches), and 152 mm (6.0 inches).

To verify the use of Equation 5 for other flat slope applications, data from Chanson and Toombes (2002) were examined and compared to the data from this study. Chanson and Toombes (2002) conducted a model study of a stepped spillway having a broad-crested weir and chute slope of 21.8°. Step heights in the model were 0.1 m (3.9 inches). Chanson and Toombes (2002) reported inception point data by referencing L_i from the upstream edge of the broad-crested weir. For this comparison, L_i from Chanson and Toombes (2002) study was transformed to start from the downstream edge of the broad-crested weir. Findings in Figure 8 illustrate that data from Chanson and Toombes (2002) agrees with the new relationship presented in Equation 5 as it was evaluated at a chute slope of 14°. Sensitivity of Equation 5 with regards to chute slope is minor, so Equation 5 evaluated at a chute slope of 22° would plot very close to what is shown on Figure 8 for a 14° slope. These results provide validity to the use of Equation 5 for determining the length from the downstream edge of a broad-crested weir to the inception point for flatter ($\theta \leq 22°$) slope stepped spillways.
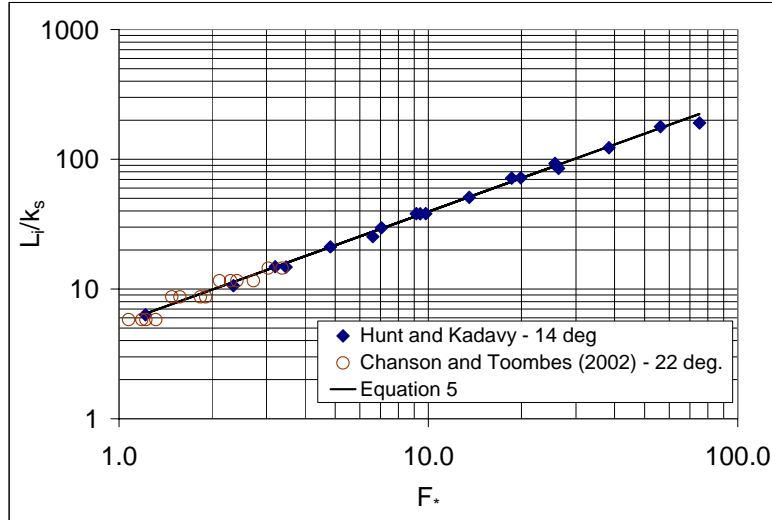
Figure 8. Normalized L_i versus Froude surface roughness, F_* for broad-crested weir stepped spillways.

Conclusions

Many earthen embankments are nearly the end of their design life, and others no longer meet state and federal dam safety guidelines due to hazard classification changes. As a result, some of these embankments are in need of rehabilitation. For those faced with hazard classification changes, increasing the spillway capacity is often required, but due to urban encroachment, land use changes, topography, and land right constraints, increasing the spillway capacity by modifying the existing dam or spillway dimensions are not always options. A popular choice for addressing these dams is the application of RCC stepped spillways over the top of the existing embankment.

The two-dimensional, physical model described herein was constructed to evaluate the inception point location in a 4(H):1(V) step spillway. The inception point location is important because it provides the engineer with the knowledge of whether to account for air entrainment in the design of the chute training walls, and it may be used for determining energy dissipation in the spillway chute. Upstream of the inception point, the flow is expected to be relatively calm. Downstream of the inception point, an aerated flow region will develop. Consequently, flow bulking, an increase in flow depth as a result of air in the flow, is anticipated downstream of the inception point. Many design criteria require complete containment of flow within the spillway chute, so if additional flow depth is expected as a result of aeration, then under current guidelines, this additional spray should be contained by the training walls.

This research provides a more optimized relationship for determining the inception point on flat sloped ($\theta \leq 22°$) stepped spillways. Research showed that Chanson's relationship effectively predicted the location of the inception point for slopes as flat as 4(H):1(V) when the Froude surface roughness, F_*, was greater than 10. In this model study, a F_* greater than 10 corresponded to step heights 38 mm (1.5 inches) and 76 mm (3.0 inches). The data was further examined and found to yield a new relationship for predicting the distance from the downstream edge of a broad-crested weir to the inception point for F_* ranging from 1 to 100. This relationship is similar to Chanson's relationship, but it was optimized for flat-sloped ($\theta \leq 22°$) stepped spillways having a broad-crested weir crest section. The findings from this research are expected to assist engineers with the design of stepped spillways applied on embankment dams.

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